ELE 251 Fundamentals of Linear Systems

Midterm 1

Feb 13, 2025

Printed Name: ___________________________

Exam Rules:

▪ Use of phones is not allowed.
▪ Electronic communication with others is not allowed.
▪ Face-to-face discussion within up to 2 groups of students is allowed.
▪ Laptop, calculator, and paper note are allowed.
▪ Text input in ChatGPT is only allowed. Other input including picture, video, document are not allowed in ChatGPT

Honor Pledge: “I affirm that I understand the exam rules and will follow them.”

Signature: ___________________________

Question 1

Perform Laplace Transform or Inverse Laplace Transform to the following:
(you can directly write down the final answer)

1.1

Perform Laplace Transform to the diff equation: $8\frac{dy(t)}{dt} + 5y(t) = 2x(t)$

1.2

Perform Laplace Transform to the signal: $x(t) = 3u(t) - 9.2\delta(t)$

1.3

Perform Laplace Transform to the ramp signal: $x(t) = 47t$

1.4

Perform inverse Laplace Transform to the signal: $Y(s) = \frac{22}{s+3}$

1.5

Perform inverse Laplace Transform and write the final result in a diff equation form:

$\frac{Y(s)}{X(s)} = \frac{-6}{s+1.4}$

Question 2

System Analysis
(you can directly write down the final answer)

2.1
When a baseball player throws a ball, the rotational motion of the ball can be analyzed using the following physical quantities:

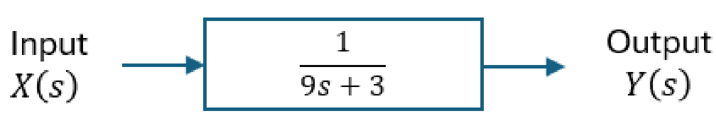
- I : is the moment of inertia.
- b : is the air resistance coefficient.
- $\omega(t)$: is the angular velocity of the ball.
- $\tau(t)$: is the torque applied to the ball by the player.

Among the listed physical quantities, from a system analysis perspective:

Which **time-dependent variable** do you define as the system input?

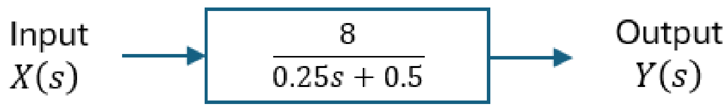
Which **time-dependent variable** do you define as the system output?

2.2
For this system, what is time constant?



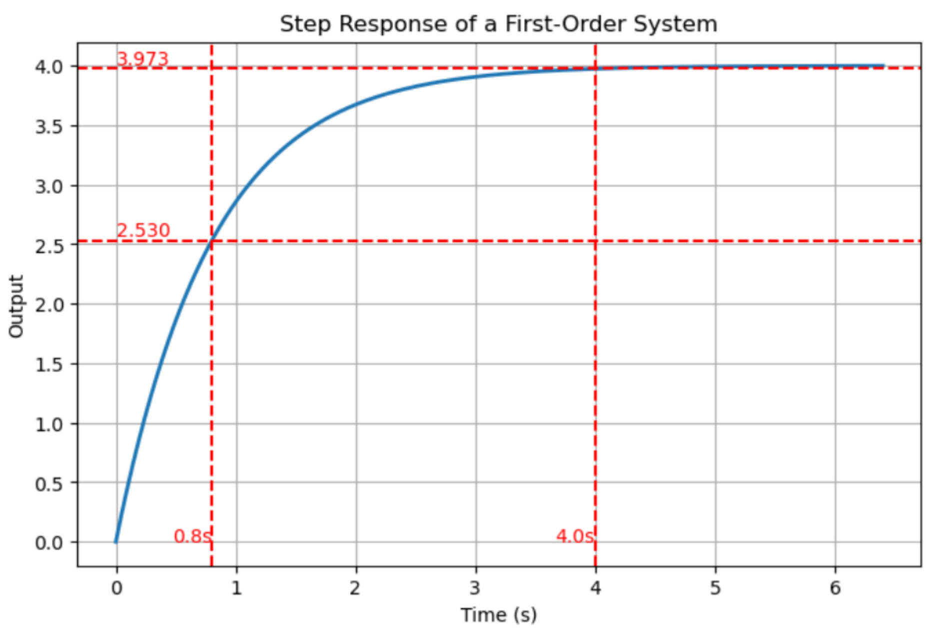
2.3

Write the transfer function in the standard form.



2.4

From this step response plot, determine the time constant of the system.



Question 3

Given a first order circuit system:

$$R_1 R_2 C \frac{dV_{out}(t)}{dt} + (R_1 + R_2)V_{out}(t) = R_2 V_{in}(t)$$

- The input and output are both voltages, units volts.
- Resistors $R_1 = 1\ \mathrm{k\Omega}$, $R_2 = 3\ \mathrm{k\Omega}$
- Capacitor $C = 1\ \mathrm{mF} = 1 \times 10^{-3}\mathrm{F}$
- Input signal $V_{in}(t) = u(t)$

Use Laplace to solve for output $V_{out}(t)$ step by step.
(In this question, you need to show detailed calculations to get full credit)

- Step 1: Transform to the Laplace domain. Also plug in numbers.

- Step 2: Solve for Output in the Laplace domain, $V_{out}(s)$
    - Show the partial fraction calculations
        - Assume Partial Fraction Form
        - Multiply by the Denominator
        - Solve for unknowns by equating coefficients

- Step 3: Inverse Laplace Transform to the time domain

Question 4

In the kitchen, there is an electric oven.

After providing constant power to the oven, the oven can be modeled as such first order system:

$$C\frac{dT(t)}{dt} = \dot{Q}_{in}(t) - hT(t)$$

- Input $\dot{Q}_{in}(t)$ is the power applied to the oven, unit W.
- Output $T(t)$ is the temperature difference between the oven and the room temperature, unit K or °C.
- C is the thermal capacity, is 3000 J/K.
- h is the heat loss coefficient, is 25 W/K

4.1
Write down the diff equation of the system with numbers plugged in.

4.2
Perform Laplace Transform to the equation.

Obtain the transfer function of the system.

4.3

Input the previous Transfer Function into Python.
Then use Python to plot the output $T(t)$ for the input of 6 kW power:

$$\dot{Q}_{in}(t) = 6000u(t)$$

Sketch the plot on the paper.
On the curve, use a "x" symbol to mark the point when is the time constant.
Please also indicate a scale for the axes.
Make sure your time-axis is properly set, so that you can see the steady state.

4.4

From your plot, determine how many **minutes** the oven takes to reach the steady state temperature?

4.5

Suppose the room temperature is 20 °C, what is the steady-state temperature of the oven in the heating? Express your result in **Fahrenheit degree °F**.

Question 5

Consider a fighter jet is launched from an aircraft carrier using a catapult system.

The fighter jet during the takeoff is the physical model to be studied.

- Input: External force, $F(t)$, that drives the fighter jet
- Output: Velocity of the fighter jet, $v(t)$

We can approximately model such system model using Newton's Second Law:

$$m\frac{dv(t)}{dt} = F(t) - bv(t)$$

Here, m (kg) is the takeoff mass of the fighter jet, b (Ns/m) is the aerodynamic drag coefficient.

5.1
The takeoff mass of the fighter jet is 12.5 tons.
The aerodynamic drag coefficient is 500 Ns/m.

Write down the model equation with numbers plugged in.

Write down the transfer function of the system.

5.2

When launched by a catapult system, the fighter jet is driven by 2 types of external force:

- Catapult Force, $F_{catapult}$: this force is 800 kN, only applied for only 1 second.
- Engine Thrust Force. F_{engine}: this force is 190 kN. It is continuously generated by the aircraft's engine.

Use impulse and step signals to represent these 2 forces.

$$F_{catapult}(t) =$$

$$F_{engine}(t) =$$

$$F(t) = F_{catapult}(t) + F_{engine}(t) =$$

5.3

Input the previous Transfer Function into Python.
Then use Python to plot the corresponding output $v(t)$.
Make sure your time-axis is properly set, so that you can see the steady state.
Write down the Python code you typed.

5.4
Sketch the plot on the paper.
Please also indicate a scale for the axes.

5.5
From your plot, determine the steady state value for the velocity $v(t)$.

Additionally, convert the steady-state velocity into miles per hour (mph).